EVALUATION OF THE MATHEMATICAL MODELS FOR QUANTIFYING THE UNSATURATED HYDRAULIC CONDUCTIVITY

EVALUAREA MODELELOR MATEMATICE DE ESTIMARE A CONDUCTIVITĂȚII HIDRAULICE

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Abstract. *Spatial an temporal variations in the soil hydraulic properties such as soil moisture, θ(h), and unsaturated hydraulic conductivity K(θ) or K(h), may affect the performance of hydrological models. In this paper, various functions are used to estimate soil hydraulic properties for a small catchment (Voinesti-Dambovita) in the western of SubCarpathians Mountain of Romania. We investigated two major issues involving soil hydraulic properties: (i) hydraulic parameter correspondence among some of the more commonly used water retention functions (i.e., the Gardner, Brooks and Corey and van Genutchen equations) and (ii) this application to estimate the unsaturated hydraulic conductivity in the vadose zone for a podzolic soil.*

Rezumat. *Variabilitatea spațială şi temporală a proprietăților hidraulice ale solului cum ar fi variația umidității în funcție de sucțiune şi variația conductivității hidraulice în mediu nesaturat, sunt folosite în modele hidraulice de calcul al fluxurilor de apă şi poluanți ce tranzitează solul. De buna lor determinare depinde acuratețea acestor modele. În acest articol sunt prezentate câteva funcții ce sunt utilizate pentru a estima proprietățile hidraulice ale unui sol de tip podzolic din subbazinul hidrologic Voineşti situat în bazinul hidrologic al Dâmboviței. Se investighează şi se prezintă rezultatele privind două proprietăți hidraulice principale: variația umidității în funcție de suctiune descrisă prin următoarele ecuații: Gardner, Brooks& Corey şi van Genutchen precum şi utilizarea rezultatelor cestora pentru a estima conductivitatea hidraulică în zona nesaturată.*

INTRODUCTION

Soil hydraulic properties: soil water retention curve and hydraulic conductivity function, are the main soil properties for determining the water retention and movement in soil.

In the literature, different methods are presented to access the hydraulic properties. Nevertheless, some these methods require expensive and very specific devices. Others needs personnel with special skills while others are very laborious and time consuming.

Moreover, the same methods are not used in all the countries and the soil type will be a reason for selecting one or another method.

Knowledge of hydraulic properties of soil is essential for irrigation and drainage design, leaching requirement formulation, and crop yield prediction. These properties are usually expressed as functional relationships between the soil hydraulic conductivity and water content or matric potential, and between the matric potential and water content (soil water retention curve). Both functions are highly nonlinear and direct laboratory or field measurements are time consuming and involve considerable uncertainty. Therefore, parameter estimation methods to determine soil hydraulic properties are of interest. Inherent in this approach is the assumption that the soil hydraulic properties may be described by a relatively simple set of deterministic function (hydraulic model) that contains few unknown parameters. The most widely used soil hydraulic models are those of Gardner (1958), which was extended by Brutsaert (1966), Brooks and Corey (1964) and van Genuchten (1980) which following Mualem models (1976).

MATERIALS AND METHODS

For modeling and other numerical or graphical purposes, it is often convenient to provide water retention and unsaturated hydraulic conductivity characteristics in functional form. For this purpose, in this paper, the Gardner, van Genuchten, Veerecken and Brooks-Corey equations are used to describe water retention characteristic with same adjustable parameters. To determine the unsaturated hydraulic conductivity we have chosen the Brutsaert and Mualem models. The equations that describe these models are presented in table 1. These models were usually based on saturated hydraulic conductivity and water retention characteristics. These are commonly given by the water content (θ, with units of cm^3 of water per cm^3 of soil) versus a capillary pressure (h, usually defined in cm of water pressure).

Table1

Mathematical models used to estimate soil water content and hydraulic conductivity

Model	Equation	Specification
Gardner and Brutsaert models (GB)	$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{\alpha}{\alpha + \lvert h \rvert^{\beta}}$, $h < 0$ $\Theta = 1$, $h \geq 0$ $K = K_S \cdot \frac{A}{A + \lvert h \rvert^{\gamma}}$	θ_r is the residual water content; θ_s is the saturated water content; K_S is the saturated hydraul conductivity; α, β, A, γ are model parameters; h is pressure head.
Brooks-Corey and Mualem models (BCM)	$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\alpha \cdot \lvert h \rvert^{\gamma}}$, if $\alpha \cdot h < 1$ $\Theta = 1$, if $\alpha \cdot h \geq 1$ $K = K_s \cdot \Theta^{k+2+\frac{2}{\lambda}}$, with $k > -1$	θ_r is the residual water content; θ_s is the saturated water content; K_S is the saturated hydraul conductivity; k is a model parameter frequent taken equal to 0,5 for Mualem model; α is an empirical parameter whos inverse is referred to the air entry value; λ is pore size distribution index; h is pressure head.
van Genuchten and Mualem models (VGM)	$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[1 + \left(\alpha \cdot \lvert h \rvert^{n}\right)\right]^{\left(1-\frac{1}{n}\right)}}$, $h < 0$ $\Theta = 1$, $h \geq 0$ $K = \frac{K_s \cdot (\alpha \cdot h)^{n-1} \cdot \left\{1 - (\alpha \cdot h)^{n-1} \cdot \left[1 + (\alpha \cdot h)^{n}\right]^{-m}\right\}^2}{\left[1 + (\alpha \cdot h)^{n}\right]^{m/2}}$	θ_r is the residual water content; θ_s is the saturated water content; K_S is the saturated hydraul conductivity; α, n are empirical constan determining the shape of the functio $n > 1$ $m = 1 - 1/n$ (Mualem, 1976); h is pressure head.

van Genuchten with Vereecken modification (VGV)	$\Theta = \dfrac{\theta - \theta_r}{\theta_s - \theta_r} = \left[1 + (\alpha \cdot \boldsymbol{h})^{\boldsymbol{n}}\right]^{-\boldsymbol{m}}$ $\theta_r = 0.015 + 0.005 \cdot \%\boldsymbol{clay} + 0.014 \cdot \%\boldsymbol{Corg}$	Corg is organic carbon content.

The soil samples was taken from the Voinesti catchment (Dambovita) from a podzolic soil. Water retention measurement in lab was determined with the pressure cell with porous plate (1 bar high flow ceramics) apparatus. The measurements was performed at -1, -4, -10, -20, -40, -70, -100, -330, and -1000 cm of capillary pressure (approximate values). Saturated hydraulic conductivity (Ks) was determined with the constant head method, after the soil samples were used in the pressure cell with porous plate apparatus.

The physic proprieties (the particle size distribution, bulk density) was also measured.

RESULTS AND DISCUTION

The soil profile is organized in 3 horizons: E (deluvial), Bt (alluvial) and C. The variation of the particle size distribution is presented in the fig. 1.

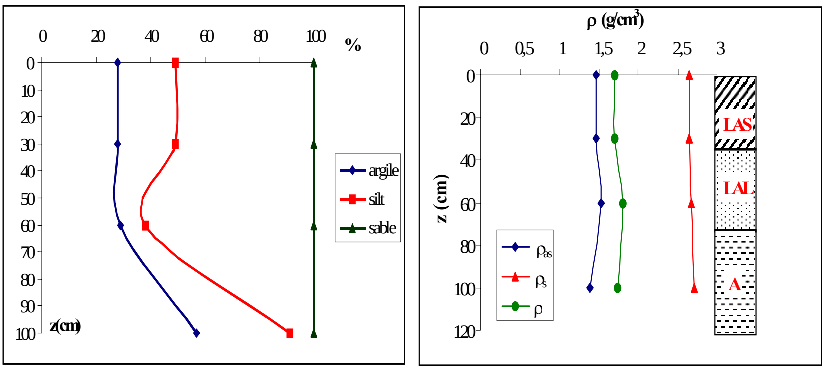

Fig. 1 - Variation of the particle size distribution and bulk density

In the upper layer of the profile, up to 60 cm depth, the content of the clay is 28-29%, but in the lower layer of the profile we observed an accumulation of clay (the clay content is 57%). The silt content varies between 9 and 34%, and the sand content is from 9% to 62%. According with the textural classification used in Romania, the soil layer up to the 30 cm can be situated in the sandy clay loess (LAS) category, the soil layer between 30 and 60 cm is a loam clay loess (LAL) and the bellow horizon (from 60 to 100 cm) enters in the clay category (A). The real density varies between 2.65 and 2.70 g/cm^3, and the bulk density varies between 1.39 and 1.53 g/cm^3. The saturated hydraulic conductivity (Ks) value varies between 1.82 and 0.22 cm/day. By following the function Ks – z (depth), we observed two stages: a fast Ks attenuation what means a fast drainage, follow by a flattening of the curve corresponding to a lent drainage.

The water retention curves for all 3 horizons are presented in fg.3. In this graphic on X-coordinate is the volumetric water content (θ) and on Y-coordinate is pF (logarithm decimal of capillarity pressure). This graphics proved that the soil presents an appreciable micro porosity in normal condition of supply water. The observed values are fitted with the equations presented in the chap. “materials and method” by minimize an objective function:

the sum of squared residuals (SSC). The suitable fit of the flexible function data is measured using root-mean-square error (RMSE) for the regression of the observed vs. fitted values ($\boldsymbol{RMSE} = \sqrt{\boldsymbol{SSC}/\boldsymbol{N}}$), the correlation coefficient (*corel*) and the Nash coefficient (*eNash*). For a suitable fit, the $\boldsymbol{RMSE} \cong 0$, $\boldsymbol{corel} \cong 1$, $\boldsymbol{and\ eNash} \cong 1\ \boldsymbol{or} \cong -1$. The parameter values are presented in table 2.

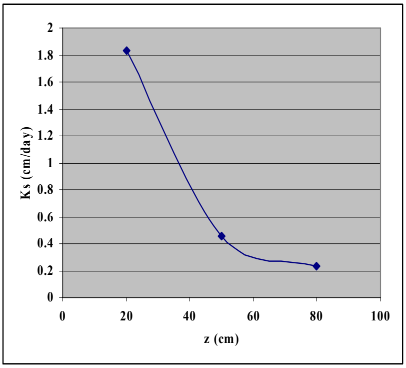

Fig. 2 - Variation Ks-z(depth)

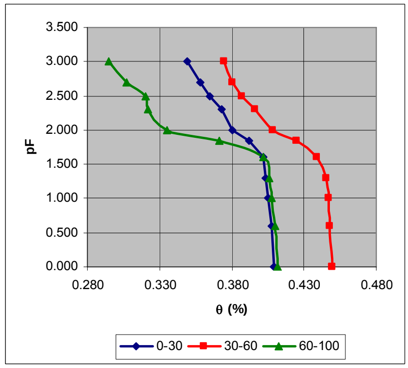

Fig. 3 - Water retention curve

Table 2

The parameter values and the values of RMSE, corel, eNash for each model and horizon

horizon	VGM		BC		GB		VGV	
0-30	α	0.009	λ	0.018	α	48.000	α	0.001
	n	1.100	α	90.000	β	0.430	n	0.930
							m	1.000
	θ_r	0.070	θ_r	0.070	θ_r	0.200	θ_r	0.110
	corel	0.981	corel	0.612	corel	0.977	corel	0.956
	RMSE	0.004	RMSE	0.029	RMSE	0.006	RMSE	0.028
	eNash	0.949	eNash	-1.196	eNash	0.900	eNash	-0.950
30-60	α	0.009	λ	0.018	α	48.000	α	0.001
	n	1.100	α	90.000	β	0.430	n	0.930
							m	1.000
	θ_r	0.070	θ_r	0.070	θ_r	0.200	θ_r	0.110
	corel	0.962	corel	0.565	corel	0.965	corel	0.928
	RMSE	0.009	RMSE	0.034	RMSE	0.009	RMSE	0.056
	eNash	0.892	eNash	-0.598	eNash	0.889	eNash	-3.388
60-100	α	0.009	λ	0.018	α	48.000	α	0.001
	n	1.100	α	90.000	β	0.430	n	0.930
							m	1.000
	θ_r	0.070	θ_r	0.070	θ_r	0.200	θ_r	0.110
	corel	0.941	corel	0.772	corel	0.947	corel	0.906
	RMSE	0.029	RMSE	0.040	RMSE	0.029	RMSE	0.019
	eNash	0.509	eNash	0.074	eNash	0.522	eNash	0.788

We observe that the models offers generally good results. The worse results are obtained with model BC for the horizon 30-60cm and 60-100cm, and with VGV model for horizon30-60cm. These results appears because we had to chose the same adjustable parameters for all soil horizons. In the case of the VGV model, the residual water content takes in a count the clay and carbonic content. Probably this variable wasn't correct estimated for which reason these models wasn't proper used to determinate hydraulic conductivity in non-saturated zone, because neither RMSE (must be close to 0) or *eNash* (must be close to +1 or –1) gives wishful results.

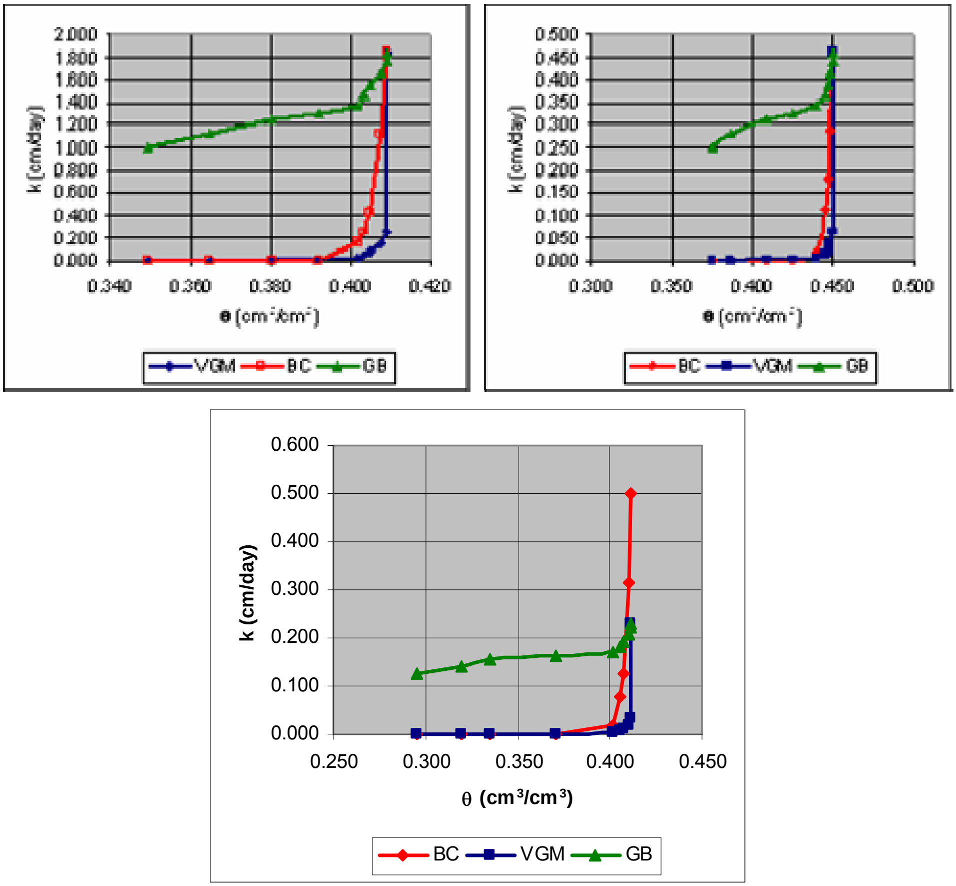

Fig. 4

The results obtained for the relation unsaturated hydraulic conductivity – water content were presented in the fig. 4. Regarding the value of the saturated hydraulic conductivity, all the used models gives good results, except the BC model for the horizon 60-100cm, for which the squared residuals is 7%. Visual, it is easy to observe that the VGM and BC models gives close results; the correlation coefficient is 0.7 for all horizons. The GB model presents a better correlation with the BG (*corel*=0.89) model than with the VGM (*corel*=0.62) model.

CONCLUSIONS

The hydraulic characteristics necessary to calculate flow and transport in porous media are water retention and saturated and unsaturated hydraulic conductivity. Four models were tested to fit the water retention curve and $k(\theta)$. Following the results obtained, we considered the VGV model doesn't offered satisfactory results, because in the calculus of the residual water content, the model use the physical soil parameters. Due to the absence of direct measurements, we had using the coefficients offered by Vereenken (see model VGV, equation to estimate the residual water content). For this reason, this model is not proper to estimate the unsaturated hydraulic conductivity. Because of default unsaturated hydraulic conductivity measurements we couldn't estimate which model is proper. However, regarding the graphics (see fig.4), the VGM and BC models offers good results for all three horizons.

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